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Enhancing the spectral sensitivity of interferometers using slow-light media

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We demonstrate experimentally that the spectral sensitivity of an interferometer can be greatly enhanced by introducing a slow-light medium into it. The experimental results agree very well with theoretical predictions that the enhancement factor of the spectral sensitivity is equal to the group index n_g of the slow-light medium. © 2007 Optical Society of America

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Slow light[1] has recently attracted tremendous attention for its promise in applications in optical communication systems, such as optical delay lines and data resynchronization. A slow-light medium is usually defined as a medium that has a very large group index as compared with the index of refraction (i.e., the phase index). Large group indices of the order of 10^7, 10^6, and 10^2 have been observed experimentally in atomic vapors,[2] room temperature solids,[3] and photonic crystal (PhC) structures,[4] respectively. Note that, although the group index n_g of PhC structures is typically smaller than that of other slow-light media, the bandwidth over which the group velocity is reduced tends to be quite large.

While many current applications of slow light take advantage of ultraslow group velocities of optical pulses inside the medium, the impact of the large dispersion inside a slow-light medium on the phase change of the optical fields has sometimes been overlooked. Moreover, interferometers with high spectral sensitivity are becoming more and more desired in applications such as metrology,[5] optical sensing,[6] quantum information processing,[7] and biomedical engineering.[8] A number of schemes have been proposed to enhance the performance of interferometers, such as using PhC structures to minimize the size of on-chip devices[9] and electromagnetically induced transparency to increase the rotation sensitivity of a Sagnac interferometer.[10]

In the present paper, we propose a novel design of spectroscopic interferometers using slow-light media. We demonstrate experimentally that the spectral sensitivity of an interferometer is proportional to the group index n_g, which can be as large as 10^7 under appropriate conditions. This enormous increase in the spectral sensitivity can be used to measure laser frequency with unprecedented precision. In practice, this increased sensitivity implies that a small change in laser frequency produces a very large change in the position of the output fringe pattern or in the output intensity of the interferometer. This effect may be important for applications such as laser metrology, the development of laser gyros, and perhaps precision tests of general relativity.

One way to see this enhancement effect is to study the output of a Mach–Zehnder (M–Z) interferometer as shown in Fig. 1. When the optical path difference between the upper and lower routes through the M–Z interferometer is caused only by a medium with length L and refractive index n, the phase difference of the two beams passing through two different routes is given by

$$\Delta\phi = \frac{\omega}{c} nL, \tag{1}$$

where ω is the optical frequency of the incident light and c is the velocity of light in vacuum. The intensity transmission of such an M–Z interferometer is given by

$$T = \frac{1}{2}(1 + \cos\Delta\phi). \tag{2}$$

Note that $\Delta\phi$ can further be a function of transverse coordinates and $\Delta\phi$ appears also in the expressions

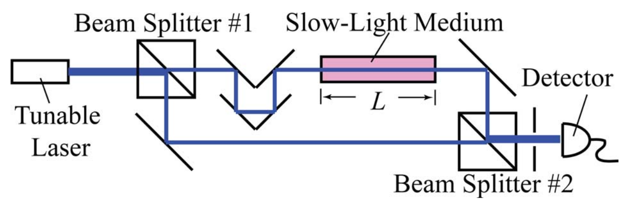

Fig. 1. (Color online) Schematic diagram of a Mach–Zehnder interferometer containing a slow-light medium in one arm.

for the transmission of many other types of interferometers.

For a spectroscopic interferometer, a change in frequency ω will produce a change in the phase difference term $\Delta\phi$, which will consequently lead to a change in intensity and enables the detection of the frequency shift. Thus, the sensitivity of a spectroscopic interferometer can typically be described in terms of the quantity $\mathrm{d}\Delta\phi/\mathrm{d}\omega$, and an interferometer is especially sensitive to a shift in frequency if the value of $\mathrm{d}\Delta\phi/\mathrm{d}\omega$ is large. Using Eq. (1), one can rewrite $\mathrm{d}\Delta\phi/\mathrm{d}\omega$ as follows:

$$\frac{\mathrm{d}\Delta\phi}{\mathrm{d}\omega} = \frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{\omega n L}{c}\right) = \frac{L}{c}\left(n + \omega\frac{\mathrm{d}n}{\mathrm{d}\omega}\right) = \frac{L n_g}{c}, \qquad (3)$$

where $n_g \equiv n + \omega \mathrm{d}n/\mathrm{d}\omega$ is the group index.

Thus, one sees that the spectral sensitivity is proportional to the group index n_g of the medium that is inside the interferometer. For common interferometers in which nondispersive media (or media with small dispersion) are used, n_g equals (or approximates) the phase index n, and therefore the dependence of spectral sensitivity on dispersion can often be neglected. However, if a slow-light medium with very large group index n_g is used, the spectral sensitivity of such an interferometer can be greatly enhanced. Alternatively, at fixed spectral sensitivity a much shorter physical path length can be used for an interferometer that uses slow light. The resulting decrease in the size of the device could be of great advantage in situations where space is limited or in which great mechanical stability is required.

To further clarify this enhancement effect, a wedged shear interferometer is analyzed. As one sees in Fig. 2, a collimated beam is normally incident on a wedge plate. Due to the interference among the light waves bouncing back and forth between the two surfaces of the wedge plate, tilt fringes occur at the exit surface. When the wedge angle θ is small (i.e., the change of the thickness of the wedge plate across the incident area of the beam is much less than its average thickness L_0), the intensity transmission as a function of lateral position y can be written as[11]

$$T(y) \approx \frac{T^2 T_L}{(1 - RT_L)^2}\frac{1}{1 + \mathcal{F}\sin^2\Delta\phi(y)}, \qquad (4)$$

where T and R are the transmittivity and reflectivity at the air–medium interface, respectively; $T_L \equiv e^{-\alpha L_0}$ is the transmittivity through the medium; $\Delta\phi = k(L_0 + \theta y) + \psi_0$ is the phase difference term; α and k are the absorption coefficient and the wavenumber of the field inside the wedge plate, respectively; ψ_0 is the phase change of field due to reflection at the air–medium interface; and $\mathcal{F}$ is the finesse defined as $\mathcal{F} \equiv 4RT_L^2/(1-RT_L)^2$. The position of the mth-order fringe peak is given by

$$y_m = \frac{(m\pi - \psi_0)c}{n\theta\omega} - \frac{L_0}{\theta}. \qquad (5)$$

If the frequency of the incident field changes slightly, the fringe peak y_m will shift laterally. The rate of movement of the fringes as a function of frequency is given by

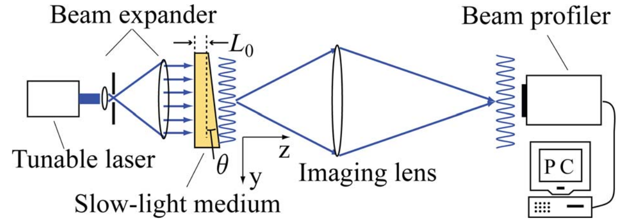

Fig. 2. (Color online) Schematic diagram of a wedged shear interferometer containing a slow-light medium.

$$\frac{\mathrm{d}y_m}{\mathrm{d}\omega} = -\frac{(m\pi - \psi_0)c}{\theta n^2\omega^2}\left(n + \omega\frac{\mathrm{d}n}{\mathrm{d}\omega}\right) \approx -\frac{\bar{m}\pi c n_g}{\theta n^2\omega^2}, \qquad (6)$$

where $\bar{m} \approx n\omega L_0/(\pi c)$ is the approximated order-number of the fringe peak (again, the small wedge angle condition of $L_0 \gg \theta y_{\max}$ is assumed). One can also normalize this movement rate with the fringe period $\Lambda = \pi c/n\omega\theta$ and obtain the following normalized expression for the rate of fringe movement:

$$\frac{1}{\Lambda}\frac{\mathrm{d}y_m}{\mathrm{d}\omega} \approx -\frac{\bar{m}n_g}{n\omega} = -\frac{L_0 n_g}{\pi c}, \qquad (7)$$

or

$$\frac{1}{\Lambda}\frac{\mathrm{d}y_m}{\mathrm{d}\lambda} \approx \frac{\bar{m}n_g}{n\lambda} = \frac{2L_0 n_g}{\lambda^2}. \qquad (8)$$

One sees from Eqs. (7) and (8) that the movement rate is proportional to the group index n_g of the wedge plate. Note that the wedge interferometer is designed to monitor the frequency shift of the incident light by detecting the lateral movement of the fringes, and therefore its spectral sensitivity is directly proportional to the rate of the fringe movement, i.e., proportional to n_g. Also note that other wavelength-dependent variables in Eq. (4), such as R, T, and $\mathcal{F}$, usually vary slowly within the wavelength range of interest, and therefore their impact on the spectral sensitivity can be ignored.

A proof-of-principle experiment has been carried out using a wedged shear interferometer. The slow-light medium used as the wedge plate in the interferometer is a $CdS_{0.625}Se_{0.375}$ c-cut single crystal. The plate is about 0.5 mm thick, and the angle between the two surfaces is about 2.28°. The absorption edge of this direct-bandgap semiconductor sample is measured to be 2.15 eV (i.e., 578 nm). According to the Kramers–Kronig relation, the refractive index of this sample experiences a rapid change near the absorption edge and consequently gives a large group index in that region. The refractive index (for ordinary polarized light) as a function of wavelength can be determined through the relation,[12] $n_o(\omega) = \sqrt{1 + 2c_0^o[y_B - q\tan^{-1}(y_B/q)]}$, where c_0^o and y_B are coefficients whose numerical values are 0.996 and 3.324 for $CdS_{0.625}Se_{0.375}$, respectively; $q \equiv \sqrt{1 - \hbar\omega/G}$; and $G = 2.15$ eV is the bandgap energy. Numerically, the values of the refractive index n and group index

n_g at $\lambda = 605$ nm are 2.64 and 3.87, respectively. As the wavelength is tuned towards the absorption edge, the group index increases much faster than the phase index. At $\lambda = 587.5$ nm, n has a value of 2.69, almost the same as that at $\lambda = 605$ nm, but the group index now is almost twice as large as n with a value of 4.80. Because the change of n and n_g as the wavelength decreases from 605 to 587.5 nm are very different, it enables us to distinguish experimentally whether the spectral sensitivity of the interferometer is dependent on n or n_g.

A Rhodamine 6G dye laser is used as the tunable source in the experiment. The fringe patterns on the exit surface of the wedge plate are imaged onto a CCD beam profiler using an imaging lens and are recorded digitally. Note that the imaging lens would not be required if the CCD were attached directly to the exit surface of the wedge plate. To determine the spectral sensitivity near each wavelength, the fringe patterns are recorded while the laser is detuned near this wavelength with a detuning step size of 0.01 nm for 21 steps. The relative peak positions are determined at each detuning step, and the fringe movement rate near this wavelength is calculated using linear fitting of the peak positions. The wavelengths at which the spectral sensitivity are measured are chosen from 605 to 587.5 nm with a decrement of 2.5 nm each time.

Figure 3 shows the relative rate of fringe movement (normalized by the fringe period) at different wavelengths. The spectral sensitivity of this interferometer increases from 10.7 to 13.5 periods/nm (see the squares in Fig. 3) as the wavelength changes from 605 to 587.5 nm. This increase agrees very well with the change of group index of $CdS_{0.625}Se_{0.375}$ as the wavelength is tuned towards the absorption edge. The solid line in Fig. 3 shows the theoretical predictions given by Eq. (8). One clearly sees that it matches very well with the experimental results. Also note that the rate of fringe movement (i.e., the spectral sensitivity) of our interferometer with a slow-light medium is about twice as large as that of an interferometer with a nondispersive medium with the same refractive index [i.e., replacing n_g by the phase index n in Eq. (8); see the dotted line in Fig. 3]. Furthermore, the sensitivity for the nondispersive interferometer changes only slightly as the wavelength changes from 605 to 587.5 nm.

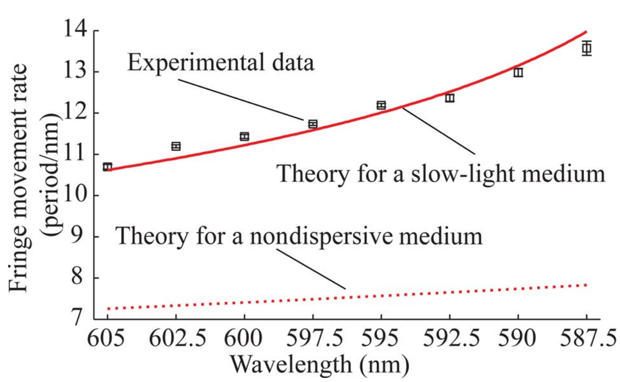

Fig. 3. (Color online) Relative rate of fringe movement as a function of wavelength. The squares are experimental data with standard deviation bars, the solid line is the theoretical prediction, and the dashed line is the prediction if dispersion is neglected.

In conclusion, we have shown that the spectral sensitivity of a interferometer scales with the group index of the material within it and hence can be greatly enhanced by introducing a slow-light medium. The enhancement effect has been demonstrated experimentally with a wedged shear interferometer based on a $CdS_{0.625}Se_{0.375}$ semiconductor material. The change of the spectral sensitivity of the interferometer at different wavelengths agrees well with the change of the group index at these wavelengths. In our proof-of-principle setup, the spectral sensitivity is enhanced by approximately a factor of 2, which is limited by the value of n_g of $CdS_{0.625}Se_{0.375}$. With an appropriate slow-light medium with a larger group index (e.g., a PhC structure or atomic vapor), the spectral sensitivity could be enhanced by a factor of 10^7 or greater. This technique can also be applied to many other types of spectroscopic interferometers, such as Mach–Zehnder, Michelson, and Fabry–Perot types.

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